



22137208



**MATHEMATICS
HIGHER LEVEL
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS**

Tuesday 21 May 2013 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematics HL and Further Mathematics SL information booklet** is required for this paper.
- The maximum mark for this examination paper is [60 marks].

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 9]

The Taylor series of \sqrt{x} about $x = 1$ is given by

$$a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3 + \dots$$

(a) Find the values of a_0 , a_1 , a_2 and a_3 . [6 marks]

(b) Hence, or otherwise, find the value of $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$. [3 marks]

2. [Maximum mark: 15]

Consider the differential equation $\frac{dy}{dx} + y \tan x = \cos^2 x$, given that $y = 2$ when $x = 0$.

(a) Use Euler's method with a step length of 0.1 to find an approximation to the value of y when $x = 0.3$. [5 marks]

(b) (i) Show that the integrating factor for solving the differential equation is $\sec x$.

(ii) Hence solve the differential equation, giving your answer in the form $y = f(x)$. [10 marks]

3. [Maximum mark: 11]

Consider the infinite series $\sum_{n=1}^{\infty} \frac{n^2}{2^n} x^n$.

(a) Find the radius of convergence. [4 marks]

(b) Find the interval of convergence. [3 marks]

(c) Given that $x = -0.1$, find the sum of the series correct to three significant figures. [4 marks]

4. [Maximum mark: 11]

(a) Express $\frac{1}{r(r+2)}$ in partial fractions. [3 marks]

(b) Let $S_n = \sum_{r=1}^n \frac{1}{r(r+2)}$.

(i) Show that $S_n = \frac{an^2 + bn}{4(n+1)(n+2)}$, where a and b are positive integers whose values should be determined.

(ii) Write down the value of $\lim_{n \rightarrow \infty} S_n$. [8 marks]

5. [Maximum mark: 14]

(a)

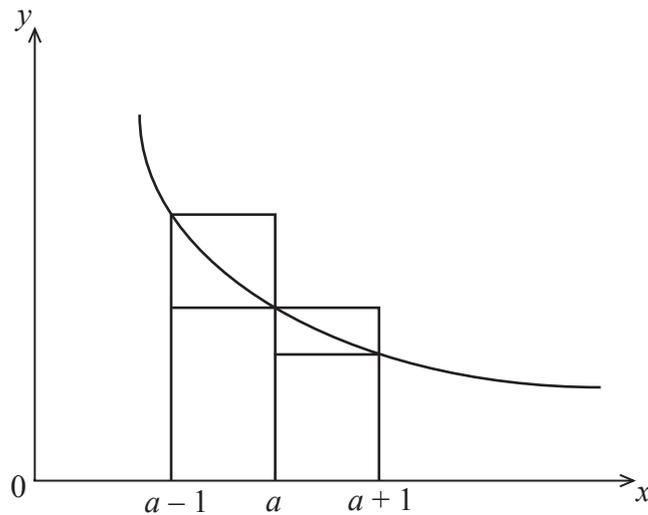


Figure 1

Figure 1 shows part of the graph of $y = \frac{1}{x}$ together with line segments parallel to the coordinate axes.

(i) By considering the areas of appropriate rectangles, show that

$$\frac{2a+1}{a(a+1)} < \ln\left(\frac{a+1}{a-1}\right) < \frac{2a-1}{a(a-1)}.$$

(ii) Hence find lower and upper bounds for $\ln(1.2)$. [9 marks]

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(Question 5 continued)

(b)

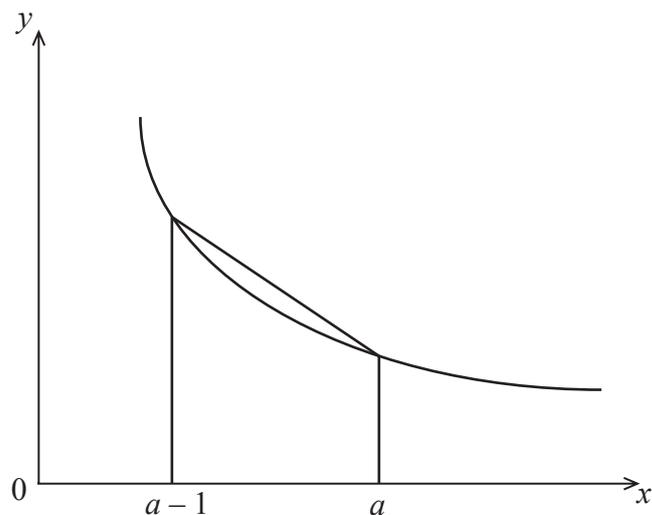


Figure 2

An improved upper bound can be found by considering Figure 2 which again shows part of the graph of $y = \frac{1}{x}$.

(i) By considering the areas of appropriate regions, show that

$$\ln\left(\frac{a}{a-1}\right) < \frac{2a-1}{2a(a-1)}.$$

(ii) Hence find an upper bound for $\ln(1.2)$.

[5 marks]